Washington Latin -- A.P. Calculus

Applications of Derivatives

INSTRUCTIONS

**→** You must do a total of 4 problems. These must include problems 1, 2, and 3.

**→** You may choose to do either problem 4 or problem 5. Problem 4 is a related rates problem and problem 5 is a maximums-minimums problem.

**→** Each of the 4 problems you do is worth 20 points, and the free response question that you did is worth 20 points.

**→** There are many details in each problem that must be considered. Make sure you consider all the details, such as units or endpoints.

**→** If you do not draw a diagram (a reasonable one) where a diagram can be drawn, I will deduct points.

**→** Otherwise, “Eli Whitney and the cotton gin.”

**SECTION A.**

You must do all problems (1 - 3) in this section.

1. WLPCS is going to build a new athletic field in the shape of a rectangle, *X* units long with semicircular regions having radius *R* at both ends. The field will have a running track on the field’s perimeter. The running track will be 400 meters in length. What values of *X* and *R* will give the largest possible area of the rectangular portion of the field?

X *track*

R

R

*not drawn to scale*

2. *f* is continuous on *[0, 3]. f, f’* and *f’’* are as follows, on the interval:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 0<x<1 | 1 | 1<x<2 | 2 | 2<x<3 | 3 |
| *f* | 0 | + | 2 | + | 0 | - | - 2 |
| *f’* | 3 | + | 0 | - | DNE | - | - 3 |
| *f’’* | 0 | - | - 1 | - | DNE | - | 0 |

“+” means positive, “-“ means negative, “DNE” means does not exist.

a) Find all extremes and determine whether they are absolute or local. Justify your conclusions. *<you might want to do part c. before a. or b.>*

b) Find all points of inflection. Justify your conclusions.

c) Sketch a possible graph of *f,* based on the information in the table.

3. Does the mean value theorem apply to the function;



a. Why?

b. If so, what value *x = c* satisfies the dictates of the mean value theorem for the function on the interval from *x = 0* to *x = 5*?

**SECTION B.**

You must do EITHER problem 4 or problem 5.

4. Use the second derivative test to find and identify all maximums and minimums of the function:



on the interval .

5. A balloon is rising vertically above a level and straight road at a constant rate of *1 ft/sec*. Just when the balloon is *65* ft above the ground, a bicycle moving at a constant speed of *17 ft/sec* passes directly under it.

a. How fast is the distance between the bicycle and the balloon increasing *3* seconds later?

b. If *θ* is the angle between the road and the line of elevation from the bicyclist to the balloon, what is the rate of change of *θ* at that same moment? [Do not bother with the calculations once you have a numerical expression for the answer.]

**EXTRA CREDIT:**